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Ghost killing in quantized Einstein–Maxwell theory

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Abstract. It is proved that the spin 2 massive ghost, with mass $\surd(-1/4b)$ arising in the non-minimal gravitational Lagrangian $\surd(-g)(R+aR^2+bR_{\mu\nu}R^{\mu\nu}+L_{\text{Maxwell}})$ cannot be decoupled from the photon field in lowest non-trivial order in perturbation theory. We conclude that gravity coupled to photons is not physically renormalizable except by a miracle.

1. Introduction

The renormalization of Einstein's theory of gravitation in the presence of matter has led to non-minimal terms in the gravitational field as necessary counter terms in the Lagrangian to remove ultraviolet divergences at single-loop level ('t Hooft and Veltman 1974, Deser and van Nieuwenhuizen 1974, Capper *et al* 1974, Nouri-Moghadam and Taylor 1975a, b). These extra terms modify the free-field propagators in such a fashion that no further counter terms are required beyond those from the single-loop contributions (Nouri-Moghadam and Taylor 1975a, b, de Witt 1964, 1967). This has led to claims (Deser 1974, de Witt 1967) that such a gravitational theory is renormalizable to all orders. But the successful treatment of the ultraviolet divergences has been achieved at the cost of ghosts introduced into the propagators to give enough damping of high-energy behaviour. This paper is a contribution to the question: should ghosts frighten you?

The ghosts present in the gravitational scalar meson theory were earlier shown (Nouri-Moghadam and Taylor 1975c, referred to as I) to be coupled to physical states of the meson–graviton system, so the ghosts could not be decoupled. The analysis was performed only at the level of certain tree graphs so that radiative corrections could still intervene to exorcise the ghosts. However such a possibility appears rather unlikely and even bordering on the miraculous since the radiative corrections to the residues of the ghost poles are non-trivial functions of the external momenta. The conclusion we reached there (I) was that the Einstein–scalar meson theory was very unlikely to be physically renormalizable.

We conclude from this that scalar mesons will not be expected to exist in a world of elementary particles in which gravitation is also taken seriously. This seems consistent with the absence of stable scalar mesons. We now have to turn to the existing particles and see if they also can exist in a gravitating world. We concentrate here on a gravitating world composed otherwise only of photons. We derive the photon–graviton vertex and investigate the residues from simple tree diagrams at the graviton–ghost vertex. As in our previous discussion we will try to discover if we can choose the various strengths of the non-minimal terms in the Lagrangian so that these residues are zero and the ghosts are exorcised. Our previous remarks on the relevance of these results to the complete set of perturbation diagrams apply also here, so that once a diagram is discovered which contributes a nonzero ghost residue the theory becomes non-physical.

2. The photon–graviton vertex

The Lagrangian density for the coupled Einstein–Maxwell theory, including the non-minimal gravitational terms, is

$$L = \sqrt{(-g)}(R+aR^2+bR_{\mu\nu}R^{\mu\nu}-\tfrac{1}{4}F_{\mu\nu}F_{\alpha\beta}g^{\mu\alpha}g^{\nu\beta}) \tag{2.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\partial_\nu = \partial/\partial x^\mu$ and A_μ is the Maxwell potential. We have already discussed the gravitational terms in (2.1) (the first three terms) and will concentrate here on the fourth term. To do that we will expand the gravitational field to first order in the quantized fluctuation $h_{\mu\nu}$ about the Minkowski background $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu}+h_{\mu\nu}$$

where the Minkowski metric is used to raise and lower the tensor indices on $h_{\mu\nu}$. Then the fourth term in (2.1) becomes

$$-\tfrac{1}{4}(1+\tfrac{1}{2}h^\alpha_\alpha)(\eta^{\mu\alpha}-h^{\mu\alpha})(\eta^{\nu\beta}-h^{\nu\beta})F_{\mu\nu}F_{\alpha\beta}. \tag{2.2}$$

We can deduce from (2.2) the vertex $V^{\alpha\beta}_{\mu\nu}$ for the coupling of a graviton with tensor components μ, ν to two photons with components α, β and momenta k_1, k_2:

$$\begin{aligned}V^{\alpha\beta}_{3\mu\nu} = {}& +\tfrac{1}{4}\{[(k_1k_2)\eta^{\alpha\beta}-k_1^{(\alpha}k_2^{\beta)}]\eta_{\mu\nu}-(k_{1\mu}k_{2\nu}+k_{1\nu}k_{2\mu})\eta^{\alpha\beta}\\ &-(k_1k_2)(\eta^\alpha{}_\mu\eta^\beta{}_\nu+\eta^\alpha_\nu\eta^\beta_\mu)+\tfrac{1}{2}(k_{1\mu}k_2^\beta+k_1^\beta k_{2\mu})\eta^\alpha_\nu+\tfrac{1}{2}(k_1^\alpha k_{2\nu}+k_{1\nu}k_2^\alpha)\eta^\beta_\mu\\ &+\tfrac{1}{2}(k_{1\mu}k_2^\alpha+k_1^\alpha k_{2\mu})\eta^\beta_\nu+\tfrac{1}{2}(k_{1\nu}k_2{}^\beta+k_1{}^\beta k_{2\nu})\eta^\alpha{}_\mu\}.\end{aligned} \tag{2.3}$$

We note that (2.3) satisfies the various gauge conditions, such as

$$k_{1\alpha}V^{\alpha\beta}_{3\mu\nu} = 0.$$

We will restrict our discussion solely to the diagram of figure 1 with the external photons on their mass shell, so that we need only consider the vertex functions

$$\begin{aligned}&\tfrac{1}{2}V^{\alpha\beta}_{\mu\nu}(e^{(1)}_\alpha(k_1)\,e^{(2)}_\beta(k_2)+e^{(2)}_\alpha(k_2)\,e^{(1)}_\beta(k_1))\\ &\quad= \mathscr{V}_{3,\mu\nu}(e^{(1)},k_1,e^{(2)},k_2)\\ &\quad= \tfrac{1}{4}\{\eta_{\mu\nu}[(k_1k_2)(e^{(1)}e^{(2)})-(e^{(1)}k_2)(e^{(2)}k_1)]-(e^{(1)}e^{(2)})(k_{1\mu}k_{2\nu}+k_{1\nu}k_{2\mu})\\ &\quad\quad-(k_1k_2)(e^{(1)}_\mu(k_1)\,e^{(2)}_\nu(k_2)+e^{(1)}_\nu(k_1)\,e^{(2)}_\mu(k_2))\\ &\quad\quad+(e^{(1)}k_2)k_{1\mu}\,e^{(2)}_\nu+(e^{(2)}k_1)k_{2\mu}\,e^{(1)}_\nu\}.\end{aligned}$$

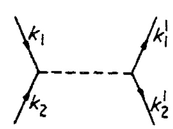

Figure 1. The diagram representing scattering of two photons with initial momenta k_1 and k_2 and final momenta k_1^1 and k_2^1 through single-graviton exchange; photons are represented by full lines, gravitons by broken lines.

3 Two-photon scattering

We will evaluate the diagram of figure 1 to be

$$\mathscr{V}_{3,\lambda\sigma}(e^{(1)1}, k_1^1; e^{(2)1}, k_2^1 Q^{\lambda\sigma\mu\nu})\mathscr{V}_{3,\mu\nu}(e^{(1)}, k_1; e^{(2)}, k_2).$$

We will take the centre of mass of the incoming system, with the pairs of photons having mutually orthogonal polarization vectors:

$$(e^{(1)} e^{(2)}) = 0.$$

Since also $(e^{(1)}k_1) = (e^{(2)}k_1) = (e^{(1)}k_2) = (e^{(2)}k_2) = 0$ then

$$\mathscr{V}_{3,\mu\nu}(e^{(1)}, k_1; e^{(2)}, k_1) = -\tfrac{1}{4}(k_1k_2)(e_\mu^{(1)}(k_1)\, e_\nu^{(2)}(k_2) + e_\nu^{(1)}(k_1)\, e_\mu^{(2)}(k_2)).$$

Thus the value of the diagram of figure 1 is

$$\tfrac{1}{4}(k_1k_2)(k_1^1k_2^1)\, e_\mu^{(1)}\, e_\nu^{(2)} Q^{\mu\nu\lambda\sigma}\, e_\lambda^{(1)1}\, e_\sigma^{(2)1}.$$

We use the results of our previous work (I) that near the spin 2 ghost pole

$$Q^{\mu\nu\lambda\sigma} \sim \frac{2}{p^2+1/4b}[(\tfrac{2}{3}\eta_{\alpha\beta}\eta_{\mu\nu} - \eta_{\alpha\mu}\eta_{\beta\nu} - \eta_{\alpha\nu}\eta_{\beta\mu}) + \tfrac{8}{3}b(\eta_{\alpha\beta}p_\mu p_\nu + \eta_{\mu\nu}p_\alpha p_\beta)$$
$$-\tfrac{64}{3}b^2 p_\alpha p_\beta p_\mu p_\nu - 4b(\eta_{\alpha\beta}p_\mu p_\nu + \eta_{\alpha\nu}p_\beta p_\mu + \eta_{\beta\mu}p_\alpha p_\nu + \eta_{\beta\nu}p_\alpha p_\mu)].$$

Then near this value of p^2

$$e_\mu^{(1)} e_\nu^{(2)} Q^{\mu\nu\lambda\sigma}\, \bar{e}_\lambda^{(1)}\, \bar{e}_\sigma^{(2)}$$
$$\sim \frac{1}{p^2+1/4b}[\tfrac{2}{3}(\bar{e}^{(1)}\, \bar{e}^{(2)})(e^{(1)}\, e^{(2)}) - (e^{(1)}\, \bar{e}^{(2)})(\bar{e}^{(1)}\, e^{(2)}) - (e^{(1)}\, \bar{e}^{(1)})(e^{(2)}\, \bar{e}^{(2)})]$$
$$= -\frac{1}{p^2+1/4b}[(e^{(1)}\, \bar{e}^{(2)})(e^{(2)}\, \bar{e}^{(1)}) + (e^{(1)}\, \bar{e}^{(1)})(e^{(2)}\, \bar{e}^{(2)})]. \tag{3.1}$$

We have that the photon momenta are

$$k_1 = (|\boldsymbol{k}_1|, \boldsymbol{k}_1), \quad k_2 = (|\boldsymbol{k}_1|, -\boldsymbol{k}_1), \quad k_1^1 = (|\boldsymbol{k}_1^1|, \boldsymbol{k}_1^1), \quad k_2^1 = (|\boldsymbol{k}_1^1|, -\boldsymbol{k}_1^1).$$

We choose that $\boldsymbol{k}_1$ is parallel to the z axis and we can further choose

$$e^{(1)} = (1, 0, 0), \qquad e^{(2)} = (0, 1, 0);$$
$$\bar{e}^{(1)} = (\cos\Theta \sin\Phi, \cos\Phi, -\sin\Theta), \qquad \bar{e}^{(2)} = (\cos\Phi, -\sin\Phi, 0)$$

with $\boldsymbol{k}_1^1 = |\boldsymbol{k}_1^1|(\sin\Theta\sin\Phi, \sin\Theta\cos\Phi, \cos\Theta)$. Thus we obtain

$$(e^{(1)}\bar{e}^{(2)})(\bar{e}^{(1)}\, e^{(2)}) = \cos\Theta\cos^2\Phi$$
$$(e^{(1)}\bar{e}^{(1)})(e^{(2)}\,\bar{e}^{(2)}) = -\cos\Theta\sin^2\Phi$$

so the residue at the spin 2 ghost pole of figure 1 is proportional to

$$(\boldsymbol{k}_1^2)^2 \cos\Theta\cos^2\Phi. \tag{3.2}$$

The value of $\boldsymbol{k}_1^2$ at the pole is $-1/16b$ so that the residue (3.2) can only vanish at the pole if b is infinite. But if we look at the form of $Q^{\mu\nu\lambda\sigma}$ in that case we obtain only the term

$$-\frac{2}{(p^2)^2}(\eta^{\mu\lambda}p^\nu p^\sigma + \eta^{\nu\lambda}p^\mu p^\sigma + \eta^{\mu\sigma}p^\nu p^\lambda + \eta^{\nu\sigma}p^\mu p^\lambda)$$

which corresponds to purely spin 1 exchange; coupling to a conserved energy-momentum tensor can only give zero contribution for all Feynman diagrams. Thus we can dispense with this case as uninteresting.

4. Discussion

We have shown that the spin 2 ghost graviton contribution to photon–photon scattering is present in the lowest-order diagram. We have already remarked that we do not expect radiative corrections to modify our result. We are thus left with the difficulty of searching elsewhere for a quantum theory of gravity which is physically sensible. However whatever else is to be added to the matter Lagrangian cannot modify at the lowest-order level at which we have been working. Only higher-order radiative corrections arising from any additional terms will be relevant, and we have already remarked that they will only cause cancellation by a miracle.

Our final conclusion is that it is not possible to quantize Einstein's theory of gravity. A similar analysis of the spinor–graviton interaction, which has been nearly completed, is needed before that conclusion is established for all forms of matter. But already at this stage we can say that we will need to look elsewhere for a quantizable theory of gravity and matter. We hope to return to this elsewhere.

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